

AN EOQ APPROACH OF NON-INSTANTANEOUS DETERIORATING ITEMS UNDER PRESERVATION AND CREDIT POLICY OF WEIBULL DISTRIBUTION, QUADRATIC DEMAND WITH SHORTAGES

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Abstract

In this paper, an inventory model for items that deteriorate over time under a price-dependent quadratic demand pattern is developed and a two-parameter Weibull distribution governs the deteriorating process. To lessen degradation and product loss, the approach includes the application of preservation technologies. A trade credit policy that the supplier provides to the retailer is also taken into account. Partially backlogged shortages are permitted. Along with the theorem proved based on economic order quantity (EOQ), sensitivity analysis, numerical examples and graph are also given to depict the model.

Keywords: Economic order quantity, Credit policy, deterioration, Weibull distribution, Quadratic demand, preservation technique.

1. Introduction

The fundamental premise of classical economic order quantity, or EOQ, is that goods are not perishable. Products are vulnerable to evaporation, dryness, obsolescence and damage in real-world situations. With increase in time, few things, such as medications, cosmetics, packaged juices, bottled soft drinks, etc., lose their uniqueness and degrade in quality. The term "non-instantaneous deterioration" describes this process. Any stored item's degradation can be avoided with the use of preservation procedures. Investing in preservation technologies lowers the pace of loss from degradation. Demand may not be linear in real-world circumstances. After reaching its peak, the demand for certain products declines. This category includes seasonal goods. The demand rate is therefore assumed to follow a quadratic function of time. Different tactics, such as discounts, resale value and permitted duration are provided in order to keep the commercial dealings with the shop. One such strategy to increase the retailer's profit is through credit policy. The supplier gives the merchant a reasonable amount of time to repay his debts without incurring interest. Because the client needs it, the store buys it right away and sells it. We refer to this as partial backlog. The degradation rate is taken to be a known constant in the majority of models. However, it is practically challenging to have an exact value. Adaptable shapes and parameters of Weibull distribution helps in capturing the complex patterns precisely. Weibull distribution is suited particularly for modeling non-instantaneous deterioration products with preservation due to its versatility and its ability to accommodate a wide range of failure and decay behaviours. In those cases, parameters of Weibull's distribution can be adjusted to reduce the rate of decay due to decay. Hence, in this paper, a retailer's optimal profit is obtained by considering the above constraints. To have a clear understanding of the topic, a number of research publications related to the paper are examined and summarised below. There are several works accessible with different kinds of demand and rates of degradation. Using three-parameter Weibull degradation as the deterioration rate, Ihsan [5] formulated an advanced method of preservation by dividing the inventory cycle in two periods. Boundary conditions were adapted in the second period at the start of deterioration to obtain optimum cycle time. According to P.K.Ghosh [9]

preservation of food depends on temperature and environmental factors. The food preparation time and preservation cycle was divided into n equal space time zones and a partial advance payment scheme was implemented. Anuj Kumar[2] calculated the profit from the retailers point of view in a fuzzy environment with shortages completely backlogged and trade credit policy. Halim[4] presented a ramp type demand problem with shortages partially backlogged and two parameter Weibull distribution as deterioration rate. The model was solved analytically and numerical example was validated. Uthayakumar [12] developed a EOQ model in which customers can return the product if they are not satisfied, but does not reimburse for the returned product with shortages partially backlogged. The returned products are resalable at the same selling price. Khyati Singh [6] advanced inventory modeling by combining a time-dependent quadratic demand function with a Weibull distribution to capture complex, non-instantaneous product deterioration patterns and developed a strategic trade credit policy into the mathematical framework, offering retailers optimized credit-period and replenishment strategies to balance holding costs against interest earnings. Under preservation technology, Mishra [8] discovered the best way to balance the overall inventory cost of decaying objects with salvage value, permitted shortages and two-parameter Weibull deterioration rate. Anu Sayal [1] concentrated on optimising his profit for a two-parameter Weibull distribution with degradation rate. Two distinct inventory models for three-parameter Weibull degradation with and without shortages were put out by Asoke [3]. A global optimisation strategy with ramp-type demand with preservation policy and two-level trade credit policy was developed by S.R. Singh [11]. Chang [7] created the best possible solution for an economical order quantity that included a cash discount and an allowable payment delay. Using degradation as a factor, Satyajit Sahu [10] created an EOQ inventory model for price-dependent demand with preservation. Consequently, a significant research gap exists in developing a unified mathematical framework that simultaneously evaluates non-instantaneous Weibull deterioration, price dependent quadratic demand, preservation technology investments and flexible financing structures under a partially backlogged shortage environment. In order to determine the best strategy for a retailer's optimal total maximum profit and replenishment cycle this research article integrates the assumed parameters and is solved by economic order quantity.

2. Assumptions & Notations

1. Deterioration rate follows two parameter Weibull distribution $R(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter, β is the shape parameter.
2. Preservation technique is included. Reduced deterioration rate due to preservation is $\phi(\pi) = 1 - e^{-\pi}$. When $\pi = 0$, $\phi(\pi) = 0$ and when $\pi \rightarrow \infty$, $\lim_{\pi \rightarrow \infty} \phi(\pi) = 1$.
3. Credit policy A is allowed to the retailer by the supplier.
4. Price dependent Quadratic demand $G(s) = a + bs + cs^2$ where $a \geq 0, b \neq 0, c \neq 0$. Here a is the initial rate of demand, b is initial rate of change of demand, c is the acceleration of the demand rate and s is the selling price of the product.
5. Lead time is zero.
6. Shortages are partially backlogged. Unsatisfied backlogged demand is $e^{-\rho(T-t)}$ with positive constant ρ as backlog parameter and $(T - t)$ waiting time, where T is the length of the inventory cycle.
7. One item is taken into study.
8. $t_0, \pi, u, C_{det}, C_{hol}$ are respectively the time when the inventory reaches zero, preservation cost, cost price of the product, deterioration cost per item and holding cost per item respectively.

3 Mathematical Formulation

Let V_n be the initial inventory level. During the period $[0, t_d]$ deterioration does not take place. The inventory level $V_1(t)$ in the period $[0, t_d]$ depends only on demand. In $[t_d, t_0]$, owing to deterioration and demand, the inventory level $V_2(t)$ at t_0 becomes zero. The deterioration in this interval is comparatively less due to

preservation technology . Shortage occurs in the period $[t_0, T]$ with inventory level $V_3(t)$. When the supplier allows the retailer a credit period A , to return his dues, three different cases are considered.

i) $0 \leq A \leq t_d$ ii) $t_d \leq A \leq t_0$ iii) $t_0 \leq A \leq T$

Differential equations of inventory $V(t)$ in interval $[0, T]$ are given by the following equations

$$\frac{dV_1(t)}{dt} = -G(s) \quad 0 \leq t \leq t_d \quad (3.1)$$

$$\frac{dV_2(t)}{dt} + \alpha \beta t^{\beta-1} (1 - \phi(\pi)) V_2(t) = -G(s) \quad t_d \leq t \leq t_0 \quad (3.2)$$

$$\frac{dV_3(t)}{dt} = -e^{-\rho(T-t)} G(s) \quad t_0 \leq t \leq T \quad (3.3)$$

Also $V_1(0) = V_n$ and $V_2(t_0) = 0$.

Solution of the above differential equations are

$$V_1(t) = V_n - G(s)t \quad (3.4)$$

$$V_2(t) = G(s) \left[1 - \alpha (1 - \phi(\pi)) t^\beta \right] \left[(t_0 - t) + \alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t^{\beta+1}}{\beta+1} \right] \quad (3.5)$$

$$V_3(t) = -G(s) \left[\frac{e^{-\rho(T-t)} - e^{-\rho(T-t_0)}}{\rho} \right] \quad (3.6)$$

Initial inventory is

$$V_n = G(s) \left[t_0 + \alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} \right] \quad (3.7)$$

Maximum backlog per cycle is

$$I_{max} = G(s) \left[\frac{1 - e^{-\rho(T-t_0)}}{\rho} \right] \quad (3.8)$$

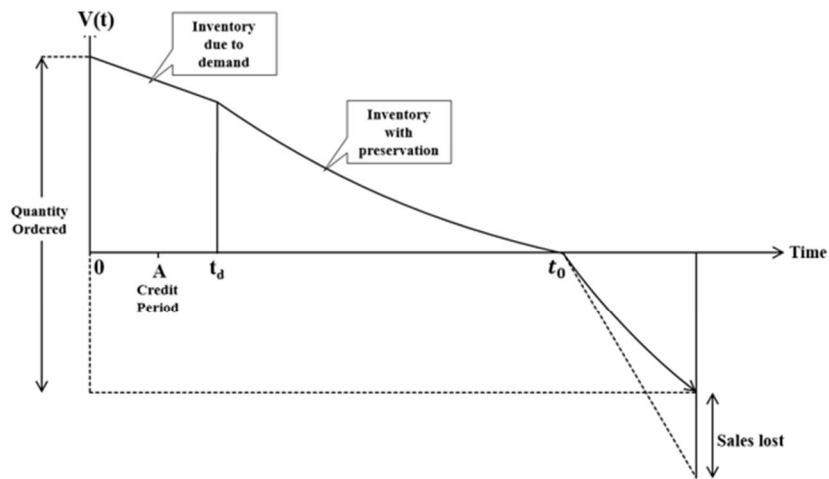


Figure 1 : Inventory when $0 \leq A \leq t_d$

Various costs associated with the inventory are as follows

1. Ordering cost per cycle is

$$CO = u (V_n + I_{max})$$

$$= u G(s) \left[t_0 + \alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} + \frac{1 - e^{-\rho(T-t_0)}}{\rho} \right] \quad (3.9)$$

2. Deterioration cost per cycle is

$$\begin{aligned} CD &= C_{det} \left[V_2(t_d) - \int_{t_d}^{t_0} G(s) dt \right] \\ &= C_{det} G(s) \left[\alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} \right] \end{aligned} \quad (3.10)$$

3. Sales lost cost is given by

$$\begin{aligned} CS &= s \int_{t_0}^T (G(s) - G(s)e^{-\rho(T-t)}) dt \\ CS &= sG(s) \left[T - t_0 + \frac{e^{-\rho(T-t_0)} - 1}{\rho} \right] \end{aligned} \quad (3.11)$$

4. Holding cost of inventory is

$$CH = C_{ho} G(s) \left[\frac{t_0^2}{2} + \alpha (1 - \phi(\pi)) t_d^{\beta+1} \left[\frac{t_0 - t_d}{\beta+1} \right] + \frac{\alpha \beta (1 - \phi(\pi))}{(\beta+1)(\beta+2)} [t_0^{\beta+2} - t_d^{\beta+2}] - \frac{\alpha^2 (1 - \phi(\pi))^2}{(\beta+1)^2} t_0^{2(\beta+1)} \right] \quad (3.12)$$

5. Preservation cost is given by

$$CP = (t_0 - t_d)\pi \quad (3.13)$$

6. Sales revenue is

$$RS = sG(s) \left[t_0 + \frac{e^{-\rho(T-t_0)} - 1}{\rho} \right] \quad (3.14)$$

7. Interest earned and Interest paid

Interest earned and paid are calculated for the three cases based on the credit period.

Case 1: $0 \leq A \leq t_d$

Interest Earned

$$IE1 = si_e \int_0^A G(s) t dt = si_e G(s) \frac{A^2}{2} \quad (3.15)$$

Interest paid

$$IP1 = ui_p \left(\int_A^{t_d} V_1(t) dt + \int_{t_d}^{t_0} V_2(t) dt \right) \quad (3.16)$$

Total Profit per unit time is

$$TP_1(t_0, \pi, T, s) = \frac{1}{T} [RS + IE1 - CO - CD - CS - CH - CP - IP1] \quad (3.17)$$

$$\begin{aligned} TP_1(t_0, \pi, T, s) &= \frac{1}{T} \left[sG(s) \left[t_0 + \frac{1 - e^{-\rho(T-t)}}{\rho} \right] + si_e G(s) \frac{A^2}{2} - u G(s) \left[t_0 + \alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} + \frac{1 - e^{-\rho(T-t_0)}}{\rho} \right] - C_{det} G(s) \left[\alpha (1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} \right] - sG(s) \left[T - t_0 + \frac{e^{-\rho(T-t_0)} - 1}{\rho} \right] - C_{hol} G(s) \left[\frac{t_0^2}{2} + \alpha (1 - \phi(\pi)) t_d^{\beta+1} \left[\frac{t_0 - t_d}{\beta+1} \right] + \frac{\alpha \beta (1 - \phi(\pi))}{(\beta+1)(\beta+2)} [t_0^{\beta+2} - t_d^{\beta+2}] - \frac{\alpha^2 (1 - \phi(\pi))^2}{(\beta+1)^2} t_0^{2(\beta+1)} \right] - (t_0 - t_d)\pi - ui_p G(s) \left[\frac{(t_0 - A)^2}{2} + \frac{\alpha (1 - \phi(\pi))}{\beta+1} [(t_0 - t_d) t_d^{\beta+1} - A(t_0^{\beta+1} - t_d^{\beta+1})] + \frac{\alpha \beta (1 - \phi(\pi))}{(\beta+1)(\beta+2)} [t_0^{\beta+2} - t_d^{\beta+2}] - \frac{\alpha^2 (1 - \phi(\pi))^2}{(\beta+1)^2} t_0^{2(\beta+1)} \right] \right] \end{aligned} \quad (3.18)$$

4. Theoretical Solution

Theorem 4.1

If $0 \leq A \leq t_d$, there exists a unique maximum solution TP_1^* for the total cost function TP_1 with respect

to inventory cycle T .

Proof :

Necessary condition for TP_1 to be maximum is $\frac{\partial TP_1}{\partial T} = 0$ and $\frac{\partial^2 TP_1}{\partial T^2} < 0$.

Differentiating (3.18) w.r.t T

$$\begin{aligned} \frac{\partial TP_1}{\partial T} = & \frac{-1}{T^2} \left[sG(s) \left[t_0 + \frac{1 - e^{-\rho(T-t)}}{\rho} \right] + si_e G(s) \frac{A^2}{2} - uG(s) \left[t_0 + \alpha(1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} + \right. \right. \\ & \left. \frac{1 - e^{-\rho(T-t_0)}}{\rho} \right] - C_{det} G(s) \left[\alpha(1 - \phi(\pi)) \frac{t_0^{\beta+1} - t_d^{\beta+1}}{\beta+1} \right] - sG(s) \left[T - t_0 + \frac{e^{-\rho(T-t_0)} - 1}{\rho} \right] - \\ & C_{hol} G(s) \left[\frac{t_0^2}{2} + \alpha(1 - \phi(\pi)) t_d^{\beta+1} \left[\frac{t_0 - t_d}{\beta+1} \right] + \frac{\alpha\beta(1 - \phi(\pi))}{(\beta+1)(\beta+2)} [t_0^{\beta+2} - t_d^{\beta+2}] - \frac{\alpha^2(1 - \phi(\pi))^2}{(\beta+1)^2} t_0^{2(\beta+1)} \right] - \\ & (t_0 - t_d)\pi - ui_p G(s) \left[\frac{(t_0 - A)^2}{2} + \frac{\alpha(1 - \phi(\pi))}{\beta+1} [(t_0 - t_d)t_d^{\beta+1} - A(t_0^{\beta+1} - t_d^{\beta+1})] + \right. \\ & \left. \frac{\alpha\beta(1 - \phi(\pi))}{(\beta+1)(\beta+2)} [t_0^{\beta+2} - t_d^{\beta+2}] - \frac{\alpha^2(1 - \phi(\pi))^2}{(\beta+1)^2} t_0^{2(\beta+1)} \right] \Bigg] + \frac{G(s)}{T} [s e^{-\rho(T-t)} - u e^{-\rho(T-t_0)} - s + \\ & s e^{-\rho(T-t_0)}] = 0. \end{aligned}$$

$\frac{\partial^2 TP_1}{\partial T^2} < 0$. Hence, TP_1 is maximum at TP_1^* . □

The following theorems can be proved for case 2 and case 3 as proved above for case 1.

Theorem 4.2

If $t_d \leq A \leq t_0$, there exists a unique maximum solution TP_2^* for the total cost function TP_2 with respect to inventory cycle T .

Theorem 4.3

If $t_0 \leq A \leq T$, there exists a unique maximum solution TP_3^* for the total cost function TP_3 with respect to inventory cycle T .

5. Solution Algorithm

Determining the second order derivatives of the total cost function with respect to the decision variables proves to be analytically complex in all three scenarios. Hence, the following algorithm is developed to find the optimal value TP_1^* , TP_2^* , TP_3^* .

Step 1 : start

Step 2 : Identify the decision variables t_0, π, T, s .

Step 3 : Assign appropriate values to the parameters.

Step 4 : Differentiate TP_i with respect to the decision variables for $i = 1, 2, 3$.

Step 5 : Equate the differentiated values to zero i.e. $\frac{\partial TP_i}{\partial t_0} = 0, \frac{\partial TP_i}{\partial \pi} = 0, \frac{\partial TP_i}{\partial T} = 0, \frac{\partial TP_i}{\partial s} = 0$ for $i = 1, 2, 3$.

Step 6 : Evaluate the Hessian matrix $H =$

$$\begin{bmatrix} \frac{\partial^2 TP_i}{\partial t_0^2} & \frac{\partial^2 TP_i}{\partial t_0 \partial \pi} & \frac{\partial^2 TP_i}{\partial t_0 \partial T} & \frac{\partial^2 TP_i}{\partial t_0 \partial s} \\ \frac{\partial^2 TP_i}{\partial \pi \partial t_0} & \frac{\partial^2 TP_i}{\partial \pi^2} & \frac{\partial^2 TP_i}{\partial \pi \partial T} & \frac{\partial^2 TP_i}{\partial \pi \partial s} \\ \frac{\partial^2 TP_i}{\partial T \partial t_0} & \frac{\partial^2 TP_i}{\partial T \partial \pi} & \frac{\partial^2 TP_i}{\partial T^2} & \frac{\partial^2 TP_i}{\partial T \partial s} \\ \frac{\partial^2 TP_i}{\partial s \partial t_0} & \frac{\partial^2 TP_i}{\partial s \partial \pi} & \frac{\partial^2 TP_i}{\partial s \partial T} & \frac{\partial^2 TP_i}{\partial s^2} \end{bmatrix}.$$

Step 7 : Negative definiteness of H proves the existence of optimum value .

Step 8 : Compare A , t_d and t_0 and go to step 9.

Step 9 : If $0 \leq A \leq t_d$, then find the optimal solution of $TP_1(t_0, \pi, T, s)$ and go to step 12. Otherwise, go to step 10.

Step 10 : If $t_d \leq A \leq t_0$,then find the optimal solution of $TP_2(t_0, \pi, T, s)$ and go to step 12. Otherwise, go to step 11.

Step 11 : If $t_0 \leq A \leq \tau$, then find the optimal solution of $TP_3(t_0, \pi, T, s)$ and go to step 12.

Step 12: Evaluate Maximum $\{TP_1(t_0, \pi, T, s), TP_2(t_0, \pi, T, s), TP_3(t_0, \pi, T, s)\}$ which gives the optimum solution TP^* of the profit

Step 13 : stop.

6. Numerical Examples

Example 6.1 :

Consider the following values if $0 \leq A \leq t_d$. Here, credit period $A = 2$, $t_d = 4$.

$C_{ord} = \$200$, $u = \$60$, $\rho = 2$, $t_0 = 8$, $\pi = \$3$, $p = \$100$, $C_{det} = \$0.5$, $s = \$2$, $i_e = 3\%$, $i_p = 4\%$, $C_{ho} = \$1$, $\alpha = 0.3$, $\beta = 0.02$.

Total profit $TP_1^* = \$1286.93$ per unit time and inventory cycle $T = 12$.

Example 6.2 :

If $t_d \leq A \leq t_0$, consider the same values as in example 6.1, except $t_d = 4$, $A = 6$, $t_0 = 8$.

Total profit $TP_2^* = \$1647.81$ per unit time.

Example 6.3 :

If $t_0 \leq A \leq T$, consider the same values as in example 6.1, except $t_d = 8$, $A = 10$, $\tau = 12$. Total profit $TP_3^* = \$1393.48$ per unit time.

Maximum total profit considering all three cases is TP_2^* , when preservation is invested.

7. Sensitivity Analysis

By changing the parameters from -20% to 20%, the total profit is calculated as sensitivity analysis and tabulated below:

Table 1 : Sensitivity Analysis of Total Profit TP_1 when $0 \leq A \leq t_d$

Change in %	π	Total Profit(\$) (by varying π)	T	Total Profit(\$) (by varying T)
-20%	0.240	1406.93	9.60	1086.93
-15%	0.255	1376.93	10.20	1136.93
-10%	0.270	1346.93	10.80	1186.93
-5%	0.285	1316.93	11.40	1236.93
0	0.300	1286.93	12.00	1286.93
5%	0.315	1256.93	12.60	1336.93
10%	0.330	1226.93	13.20	1386.93
15%	0.345	1196.93	13.80	1436.93
20%	0.360	1166.93	14.40	1486.93

8. Graphical Illustrations

Based on example and sensitivity analysis, the following graphs are illustrated to visualize the effect of decision variables on the total profits.

Figure 2

Effect of Preservation cost π on total profits

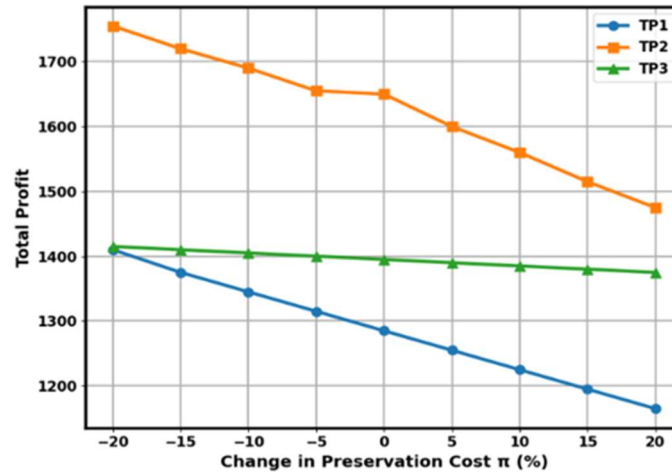
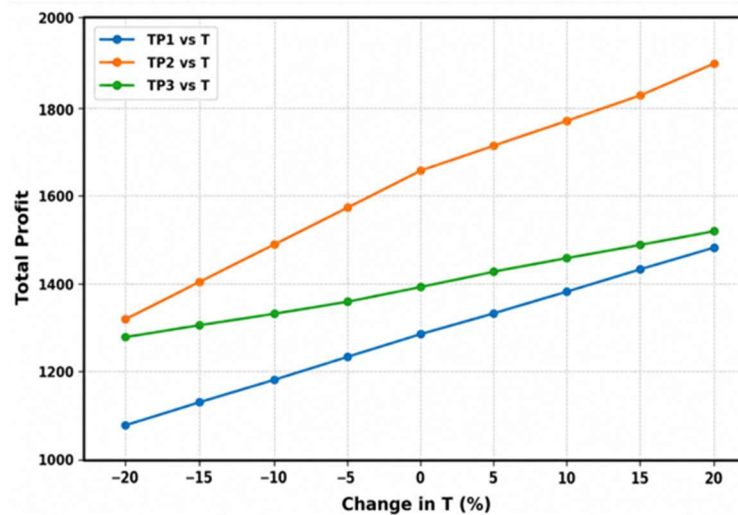


Figure 3

Effect of inventory cycle T on total profits



9. Key Insights

Based on the sensitivity tables and figures for case 1, 2 and 3 the following insights are observed.

1. Comparison of Overall profits: From three examples, Case 2 consistently yields the highest profit among all the three cases, due to preservation and is the optimal value of profit of the model developed.

2. Preservation cost π : Figure 2 depicts the trend of preservation cost. As π increases from

-20% to 20% , the total profits decreases. Total profit TP_1 is most affected, suggesting higher decline due to increase in preservation cost. TP_3 shows least decline indicating some resilience due to the trade credit. Hence, higher preservation costs negatively impact profits. Managing preservation investment cost effectively is crucial.

3.Inventory cycle time T : As inventory cycle increases, deterioration cost rises. TP_1 decreases slowly with higher T , due to higher deterioration cost. In figure 3, TP_2 shows moderate decline and TP_3 shows least reduction, due to credit effort. Beyond a certain point, increasing τ causes profits to decline.

10. Conclusion

In this price dependent quadratic model of Weibull deterioration rate with preservation and shortages , it is observed that extending both the preservation period and the credit period significantly boosts profit. However, an increase in preservation costs leads to a decline in profit , emphasizing the importance of cost-effective preservation management. Extending the preservation period beyond an optimal point yields minimal additional profit. Hence a strategic policy that extends preservation time, controlling preservation cost and balanced credit terms can significantly enhance overall profitability.

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